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# Complex-Vector PLL for Enhanced Synchronization with Weak Power Grids

## Abstract

This paper proposes a complex-vector phase-locked loop (PLL) that can eliminate the frequency-coupling terms caused by the asymmetric dynamics of conventional PLL. In the approach, a complex phase angle with both real and imaginary components is introduced, which enables to control the direct and quadrature axis components symmetrically, and thus the original multiple-input multiple-output (MIMO) system can be simplified to single-input single-output (SISO) complex transfer functions, which greatly simplifies the system stability analysis. Moreover, the dangerous subsynchronous oscillation hazard with the conventional PLL can be avoided, and the SISO impedance shaping technique can be utilized to enhance the grid synchronization stability under the weak grid condition. The complex-vector PLL is verified by both the theoretical analysis and experimental tests.

## I. Introduction

Over the past decade, renewable energy based distributed power generations have been continuously integrated into power grids. More and more power electronic components are being used as the interface in the renewable power plants and distribution networks. Despite great advantages of power electronics in terms of high controllability and improved efficiency, they may also bring in various of resonance and stability issues due to the interactions between the wideband control loops, the passive power filters, as well as the grid impedance [1].

To effectively address these challenges, the impedance-based approach has been recently developed in [2], which not only provides an intuitive insight of the interactions among the power converters but also enables to reshape output impedance for stabilizing the power system. With this method, a converter, including the whole control system, and the passive filters as an output impedance connected with a voltage or current source. Depending on the chosen frames, different linearization methods have to be utilized to establish impedance model of the converter, resulting in the dq-domain impedance model [3] and the sequence-domain impedance model [4]. The mathematical relations between the two models are established in [5], and it is revealed that all the models should have the same stability implications.

In order to characterize the dynamics of the phase-locked loop (PLL), matrix-based impedances model has to be used to represent the frequency coupling effects induced by the inherent asymmetry of PLL [5]. As a result, the General Nyquist Criterion (GNC) should be utilized for assessing the stability of this multi-input multi-output (MIMO) system. Many efforts have been made to approximate the MIMO system to single-input and single-output (SISO) system for simplification [6]. However, as emphasized in [7], even though the couplings are small in magnitude, neglecting them results in a wrong estimation of system stability. Moreover, the frequency coupling effect tends to introduce a sideband oscillation below the fundamental frequency, which can easily propagate in the power system and may trigger the dangerous subsynchronous oscillations (SSO) together with the traditional power plant [5].

In this paper, the complex-vector phase-locked loop (PLL) is proposed, which can remove the frequency-coupling issues directly by tracking the complex phase angle, i.e., the phase angle with both the real part and imaginary part. In this way, the PLL is symmetric by nature and can be represented by SISO impedance model, which can greatly simplify stability analysis and eliminate the SSO hazard caused by PLL. Meanwhile, the impedance shaping can be easily implemented to enhance the grid synchronization stability under the weak grid conditions.

## II. Control Scheme of the Complex-Vector PLL

For good comparison, the control scheme of the conventional PLL is depicted in Fig. 1, where the input of PLL is merely the  $q$ -axis voltage. Hence, this asymmetric control structure will introduce frequency couplings when the dynamics of PLL are included, and asymmetric transfer matrices with the cross-couplings between the  $d$ - and  $q$ -axis components are inevitable [4], which will introduce the frequency coupling effects. To address this issue, the complex-vector PLL is proposed, of which the control scheme is shown in Fig. 2.

Different from the conventional PLL, the input of complex vector PLL contain both  $d$ -axis and  $q$ -axis voltages. The output of the complex vector PLL is also the complex phase angle  $\theta$ , which includes both the  $d$  and  $q$  components. Hence the control structure is symmetric which can remove the frequency couplings. As seen, in order to use the complex phase angle  $\theta$  to do dq transformation, the transformation rule should be modified accordingly. In complex vector form, it can be expressed as:

$$\mathbf{v}_{dq}^c = e^{-j\theta} \cdot \mathbf{v}_{\alpha\beta} = e^{-j(\theta_d + j\theta_q)} \cdot \mathbf{v}_{\alpha\beta} \quad (1)$$

In real vector form, it can also be expressed as:

$$\begin{bmatrix} v_d^c \\ v_q^c \end{bmatrix} = e^{\theta_q} \begin{bmatrix} \cos \theta_d & \sin \theta_d \\ -\sin \theta_d & \cos \theta_d \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \quad (2)$$

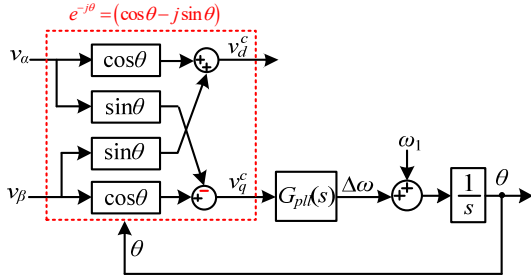


Fig. 1. Control scheme of the conventional PLL

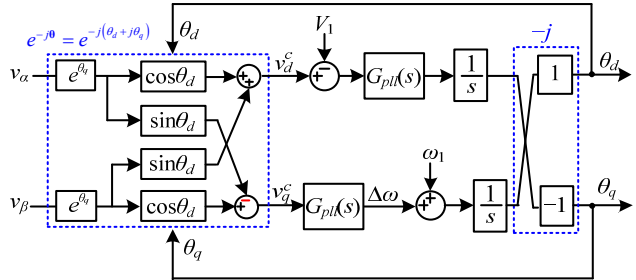


Fig. 2. Control scheme of the complex-vector PLL

## III. Small-signal Modeling

For better illustration, two  $dq$  frames are defined in this paper. One is the grid  $dq$  frame that defined by the phase angle of fundamental positive-sequence grid voltage  $\mathbf{v}_g$ , denoted as  $\theta_1$ . The other is the converter  $dq$  frame, which is defined by the phase angle obtained from PLL, denoted as  $\theta$ . Vectors in the converter  $dq$  frame will be denoted with the superscript  $c$ . Given a perturbation on the grid voltage in the grid  $dq$ -frame,  $\Delta \mathbf{v}$ , the grid voltage in the stationary  $\alpha\beta$  frame can be expressed as

$$\mathbf{v}^s = v_\alpha + jv_\beta = (V_1 + \Delta \mathbf{v}) e^{j\theta_1} \quad (3)$$

where  $V_1$  is the steady-state grid voltage aligned to the  $d$ -axis, i.e. the steady-state grid voltage vector,  $\mathbf{V}_1 = V_1 + j0$ . The response of the detected complex phase  $\theta$  corresponding to the voltage perturbation can be given as

$$\theta = \theta_1 + \Delta \theta \quad (4)$$

Thus, the output voltage of the  $dq$  transformation is derived as below:

$$\mathbf{v}^c = \mathbf{v}^s e^{-j\theta} = (V_1 + \Delta \mathbf{v}) e^{-j\Delta \theta} \approx (V_1 + \Delta \mathbf{v})(1 - j\Delta \theta) \Rightarrow \Delta \mathbf{v}^c = \mathbf{v}^c - \mathbf{v}_1^c \approx \Delta \mathbf{v} - jV_1 \Delta \theta \quad (5)$$

Then, considering the PLL controller  $G_{pll}(s)$ , the detected phase variation  $\Delta \theta$  is given by

$$\Delta\theta = -jG_{pll}(s) \cdot \frac{1}{s} \cdot \Delta\mathbf{v}^c \quad (6)$$

Substituting (6) into (5), yields

$$\Delta\theta = -j \frac{G_{pll}(s)}{s + G_{pll}(s)V_1} \Delta\mathbf{v} \quad (7)$$

(7) clearly shows the PLL phase angle dynamics with respect to a small signal grid voltage variation.

By substituting (7) back into (5), the relationship between the grid voltage variations in the converter dq frame and grid dq frame can be obtained by:

$$\Delta\mathbf{v}^c = \Delta\mathbf{v} - jV_1\Delta\theta = \Delta\mathbf{v} - jV_1 \cdot \left( -j \frac{G_{pll}(s)}{s + G_{pll}(s)V_1} \Delta\mathbf{v} \right) = \frac{s}{s + G_{pll}(s)V_1} \Delta\mathbf{v} \quad (8)$$

$$\underbrace{\frac{s}{s + G_{pll}(s)V_1}}_{H_{PLL1}(s)}$$

Similarly, the injected grid current in the stationary  $\alpha\beta$  frame can be given by:

$$\mathbf{i}^s = \mathbf{i}^c e^{j\theta} = \mathbf{i} e^{j\theta_i} \quad (9)$$

So the grid currents in the converter dq frame can be derived as:

$$\mathbf{i}^c = \mathbf{i}^s e^{-j\theta} = (\mathbf{i}_1 + \Delta\mathbf{i}) e^{-j\Delta\theta} \approx (\mathbf{i}_1 + \Delta\mathbf{i})(1 - j\Delta\theta) \Rightarrow \Delta\mathbf{i}^c = \mathbf{i}^c - \mathbf{i}_1 \approx \Delta\mathbf{i} - j\mathbf{i}_1\Delta\theta \quad (10)$$

where  $\mathbf{i}_1$  is the steady-state grid current in converter dq frame, which is also equal to steady-state grid current in the grid dq frame.

By substituting (7) to (11), the small signal-perturbation of the grid current in the converter dq frame can be derived as:

$$\Delta\mathbf{i}^c \approx \Delta\mathbf{i} - j\mathbf{i}_1\Delta\theta = \Delta\mathbf{i} - \underbrace{\frac{G_{pll}(s) \cdot \mathbf{i}_1}{s + G_{pll}(s)V_1}}_{H_{PLL2}(s)} \Delta\mathbf{v} \quad (11)$$

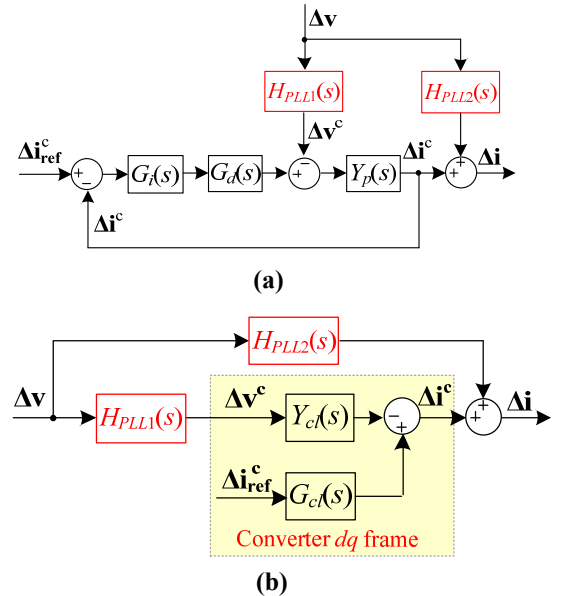
According to (8) and (11), the control block diagram of the grid-connected converter considering the complex vector PLL dynamics is shown in Fig. 3 (a), where  $G_f(s)$  is the current controller,  $G_d(s)$  is 1.5 times of switching period digital control delay that introduced by the computation and PWM modulation,  $Y_p(s)$  is the admittance of the converter-side inductor, expressed as  $Y_p(s) = 1/(s + j\omega_1)L$ . The closed-loop response of the current control system can thus be given by

$$\mathbf{i}^c = \underbrace{\frac{T(s)}{1+T(s)}}_{G_{cl}(s)} \mathbf{i}_{ref}^c - \underbrace{\frac{Y_p(s)}{1+T(s)}}_{Y_{cl}(s)} \mathbf{v}^c \quad (12)$$

where  $T(s)$  is the open loop gain of the current control, given by:

$$T(s) = G_i(s)G_d(s)Y_p(s) \quad (13)$$

Accordingly, Fig. 3(a) can be simplified as shown in Fig. 3 (b). As seen, the complex vector PLL affects converter output admittance in two ways: (1) change the closed-



**Fig. 3. Block diagram of the grid converter considering the dynamics of complex vector PLL (a) original diagram (b) equivalent diagram**



algorithms are programmed in the dSPACE1007. The circuit parameters are shown in Table I, where the grid voltage is intentionally reduced to create the weak grid condition with  $SCR = 1.5$ . Using the conventional PLL, the converter becomes unstable and soon triggered off when the crossover frequency of the PLL is set at 30Hz, as shown in Fig 5(a). However, when the proposed complex-vector PLL with impedance shaping is used, the system becomes stable for the same control parameters, as shown in Fig 5(b). More experimental results will be presented in the full paper.

TABLE I PARAMETERS OF POWER CONVERTER

Parameter	Values
$V_{dc}$	Input dc-link voltage
$V_1$	Rated line to line grid voltage, RMS
$f_1$	Grid fundamental frequency
$f_{sw}$	Inverter switching frequency
$f_s$	Inverter control sampling frequency
$P_n$	Rated power
$L_1$	Inverter-side inductor
$C_f$	Filter capacitor
$L_t$	Equivalent grid-side inductor( $L_2+L_g$ )
$SCR$	Short Circuit Ratio

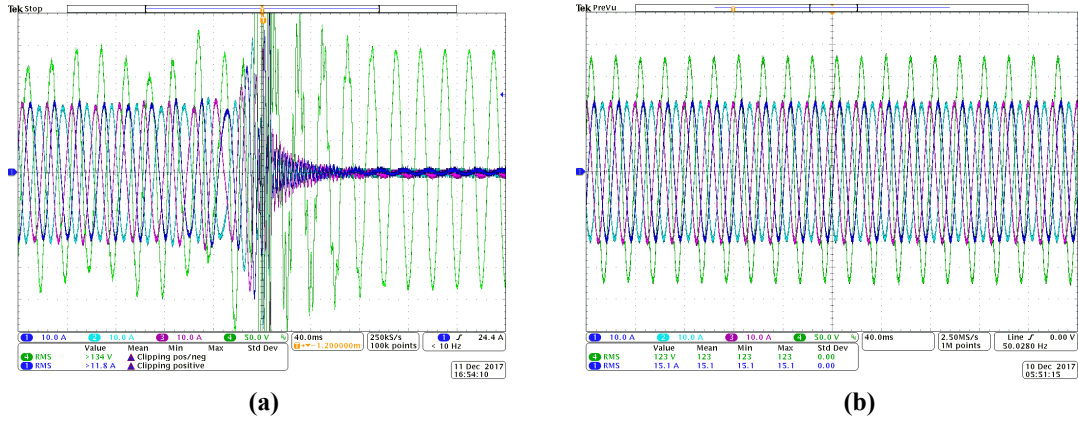


Fig. 5. Dynamic waveforms with a step change of active power reference between 0.6 p.u. to 1 p.u.

## V. Conclusions and Future Work

In this paper, the complex-vector PLL is proposed to remove the frequency coupling issues. The small signal modeling of the complex-vector PLL dynamics is established, which can be represented by the SISO complex transfer functions. Based on which, the adverse impact of the PLL is investigated from the view of physical interpretation, and the impedance shaping method is proposed to enhance the grid synchronization stability under the weak grid condition. The experimental results verify the effectiveness of the proposed method. More stability analysis and experimental results will be presented in full paper.

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